A domain theory for statistical probabilistic programming

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with
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Statistical probabilistic programming

$$[\![-]\!] : \text{programs} \to \text{distributions}$$

- Continuous types: $\mathbb{R}, [0,\infty]$
- Probabilistic effects:

normally distributed sample

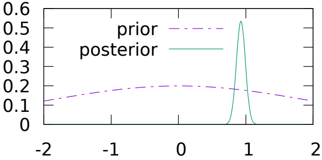
$$\mathbf{sample}(\mu,\sigma) : \mathbb{R}$$

scale distribution by r

$$\frac{r : [0,\infty]}{\mathbf{score}(r) : \mathbf{1}}$$

conditioning/fitting to observed data

$$[\![\mathbf{sample}(0,2)]\!]$$



prior

$$[\![\begin{array}{l} \mathbf{let}\ x{=}\mathbf{sample}(0{,}2) \\ \mathbf{in}\ \mathbf{score}(normalPdf(1.1|\ x,\tfrac{1}{4})); \\ \quad \mathbf{score}(normalPdf(1.9|2x,\tfrac{1}{4})); \\ \quad \mathbf{score}(normalPdf(2.7|3x,\tfrac{1}{4})); \\ \quad x \end{array}]\!]$$

posterior

Statistical probabilistic programming

Exact Bayesian inference using disintegration [Shan-Ramsey'17]

- Commutativity/exchangability/Fubini

$$\left[\!\!\left[\begin{array}{l}\textbf{let}\ x = K\ \textbf{in}\\ \textbf{let}\ y = L\ \ \textbf{in}\\ f(x,y)\end{array}\right]\!\!\right] = \left[\!\!\left[\begin{array}{l}\textbf{let}\ y = L\ \ \textbf{in}\\ \textbf{let}\ x = K\ \textbf{in}\\ f(x,y)\end{array}\right]\!\!\right] \qquad \begin{array}{c}\int [\![K]\!](\mathrm{d}x)\int [\![L]\!](\mathrm{d}y) f(x,y)\\ =\\ \int [\![L]\!](\mathrm{d}y)\int [\![K]\!](\mathrm{d}x) f(x,y)\end{array}$$

probabilitity distributions ✔

σ-finite distributions ✔

arbitrary distributions ✗

s-finite distributions ✔

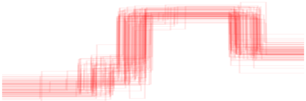
not closed under push-forward

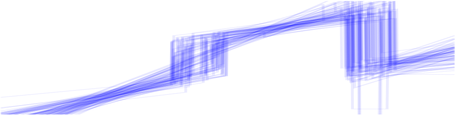
full definability [Staton'17]

Statistical probabilistic programming

Express continuous distributions using:

- Higher-order functions





piecewise(random-constant) *piecewise(random-linear)*

example: generative random function models

[Heunen et al.'17]

measure theory ✗

measurable cones and stable measurable functions ✓

quasi-Borel spaces ✓

[Ehrhard-Pagani-Tasson'18]

Theorem (Aumann'61)

No σ-algebra over $\mathbf{Meas}(\mathbb{R}, \mathbb{R})$ *with measurable evaluation:*

$$\text{eval} : \mathbf{Meas}(\mathbb{R}, \mathbb{R}) \times \mathbb{R} \to \mathbb{R}$$

Statistical probabilistic programming

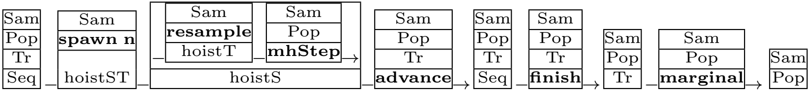
Express continuous distributions using:

- Inductive types and bounded iteration

Ścibior et al.'18a+b

resamples particles moves

```
rmsmc k n t =
  marginal . finish . compose k (
    advance . hoistS (
      compose t mhStep . hoistT resample
    )
  ) . hoistST (spawn n >>)
```



see Adam Ścibior's talk

Monday 17:02–17:25

Functional Programming for Modular Bayesian Inference

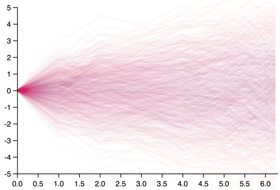
Statistical probabilistic programming

Express continuous distributions using:

[Ehrhard-Pagani-Tasson'18]

- Term recursion

$$
\begin{aligned}
&rw(x,\sigma) = \lambda(). \qquad \text{// thunk}\\
&\quad \mathbf{let}\, y = \mathbf{sample}(x,\sigma)\\
&\quad \mathbf{in}\,(x, rw\,(y,\sigma))
\end{aligned}
$$



Gaussian random walk

- Type recursion and dynamic types

Church *Web***PPL** Venture

this talk

Iso-recursive types: FPC

type variable contexts $\Delta = \{\alpha_1, \ldots, \alpha_n\}$

[Fiore-Plotkin'94]

$$\frac{\Delta, \alpha \vdash_{\mathrm{k}} \tau : \mathsf{type}}{\Delta \vdash_{\mathrm{k}} \mu\alpha.\tau : \mathsf{type}}$$

$$\begin{aligned} Lam = \ & \mu\alpha.\{\mathrm{Bool}\{\mathrm{True} \mid \mathrm{False}\} \\ & \mid \mathrm{App}(\alpha * \alpha) \\ & \mid \mathrm{Abs}(\alpha \to \alpha)\} \end{aligned}$$

type recursion

$\tau = \mu\alpha.\sigma$

$$\frac{\Gamma \vdash t : \sigma[\alpha \mapsto \tau]}{\Gamma \vdash \tau.\mathbf{roll}\,(t) : \tau} \qquad \frac{\Gamma \vdash t : \tau \qquad \Gamma, x : \sigma[\alpha \mapsto \tau] \vdash s : \rho}{\Gamma \vdash \mathbf{match}\, t \,\mathbf{with}\, \mathbf{roll}\, x \Rightarrow s : \rho}$$

Iso-recursive types: FPC

[Fiore-Plotkin'94]

type variable contexts
$\Delta = \{\alpha_1, \ldots, \alpha_n\}$

$$\frac{\Delta, \alpha \vdash_{\mathrm{k}} \tau : \mathsf{type}}{\Delta \vdash_{\mathrm{k}} \mu\alpha.\tau : \mathsf{type}}$$

$\omega\mathbf{Cpo}$-enriched category of domains

type recursion

$$[\![\Delta \vdash_{\mathrm{k}} \tau : \mathsf{type}]\!] : (\mathcal{C}^{\mathrm{op}})^n \times \mathcal{C}^n \to \mathcal{C}$$

$$[\![\Delta \vdash_{\mathrm{k}} \mu\alpha.\tau : \mathsf{type}]\!] = \text{minimal invariants}$$

[Freyd'91,92, Pitts'96]

locally continuous functor

Challenge

- probabilistic powerdomain
- commutativity/Fubini
- domain theory
- higher-order functions

continuous domains [Jones-Plotkin'89]

open problem [Jung-Tix'98]

traditional approach:

domain $\mapsto$ Scott-open sets $\mapsto$ Borel sets $\mapsto$ distributions/valuations

our approach:

as in [Ehrhard-Pagani-Tasson'18]

$$(\text{domain}, \text{quasi-Borel space}) \mapsto \text{distributions}$$

separate but compatible

Summary

Contribution

- $\omega\mathbf{Qbs}$: a category of pre-domain quasi-Borel spaces
- M: commutative probabilistic powerdomain over $\omega\mathbf{Qbs}$

Theorem (adequacy)

M adequately interprets:

- *Statistical FPC*
- *Untyped Statistical λ-calculus*

This talk

- $\omega\mathbf{Qbs}$
- a powerdomain over $\omega\mathbf{Qbs}$
- a domain theory for $\omega\mathbf{Qbs}$

Rudimentary measure theory

Borel sets

- $[a, b]$ Borel
- A Borel $\implies$ $A^{\complement}$ Borel
- $(A_n)_{n\in\mathbb{N}}$ Borel $\implies$ $\bigcup_{n\in\mathbb{N}} A_n$ Borel

Measurable functions $f : \mathbb{R} \to \mathbb{R}$

$f^{-1}[A]$ Borel $\Longleftarrow$ A Borel

Measures μ : Borel $\to [0, \infty]$

- monotone:
  $A \subseteq B \implies \mu(A) \leq \mu(B)$
- Scott-continuous:
  $A_0 \subseteq A_1 \subseteq \ldots \implies$
  $\mu\left(\bigcup_n A_n\right) = \bigvee_n \mu(A_n)$

Example

Lebesgue measures:

$\lambda[a, b] = b - a$ on $\mathbb{R}$

$(\lambda \otimes \lambda)([a, b] \times [c, d]) = (b - a)(d - c)$ on $\mathbb{R}^2$

1 dimensional

2 dimensional

Push-forward measure

$f_*\mu(A) := \mu\left(f^{-1}[A]\right)$

Borel set

measure

$f : \mathbb{R} \to \mathbb{R}$

Quasi-Borel pre-domains

set

partial order on $\underline{X}$

subset of functions
$\mathbb{R} \to \underline{X}$
"random elements"

ω-qbs:

$$X = (\underline{X}, \leq_X, M_X)$$

Quasi-Borel pre-domains

ω-qbs:

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"random elements"

$$X = (\underline{X}, \leq_X, M_X)$$

ω-cpo

- $x_0 \leq x_1 \leq x_2 \leq \ldots \qquad \Longrightarrow \qquad \exists \bigvee_n x_n$

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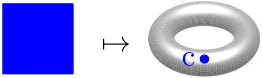
ω-cpo

quasi-Borel
space

- $\lambda\_.x \in M_X$



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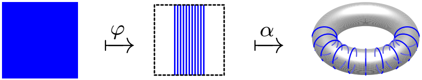
quasi-Borel
space

$\mathbb{R} \xrightarrow{\varphi}_{\text{Borel}} \mathbb{R}$

- $\lambda\_.x \in M_X$
- $\alpha \in M_X \implies \alpha \circ \varphi \in M_X$



Quasi-Borel pre-domains

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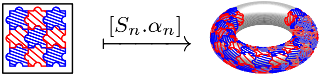
$\mathbb{R} \xrightarrow{\varphi}_{\text{Borel}} \mathbb{R}$

- $\lambda\_.x \in M_X$
- $\alpha \in M_X \implies \alpha \circ \varphi \in M_X$
- $(\alpha_n \in M_X)_{n\in\mathbb{N}} \implies [r \in S_n.\alpha(r)] \in M_X$



Borel measurable
countable partition
$\mathbb{R} = \biguplus_{n\in\mathbb{N}} S_n$

Quasi-Borel pre-domains

ω-qbs:

set; partial order on $\underline{X}$; subset of functions $\mathbb{R} \to \underline{X}$ "random elements"

$$X = (\underline{X}, \leq_X, M_X)$$

ω-cpo; quasi-Borel space

- $\lambda\_.x \in M_X$
- $\alpha \in M_X \implies \alpha \circ \varphi \in M_X$ ($\mathbb{R} \xrightarrow{\varphi}_{\text{Borel}} \mathbb{R}$)
- $(\alpha_n \in M_X)_{n\in\mathbb{N}} \implies [r \in S_n.\alpha(r)] \in M_X$

Borel measurable countable partition $\mathbb{R} = \biguplus_{n\in\mathbb{N}} S_n$

s.t.:

pointwise ω-chain; pointwise lub

$$(\alpha_n) \in M_X^\omega \implies \bigvee_n \alpha_n \in M_X$$

Quasi-Borel pre-domains

ω-qbs:

$$X = (\underline{X}, \leq_X, M_X)$$

- set: $\underline{X}$
- partial order on $\underline{X}$: $\leq_X$
- subset of functions $\mathbb{R} \to \underline{X}$ "random elements": M_X
- ω-cpo: $(\underline{X}, \leq_X)$
- quasi-Borel space: $(\underline{X}, M_X)$

- $\lambda\_.x \in M_X$
- $\alpha \in M_X \implies \alpha \circ \varphi \in M_X$ (where $\mathbb{R} \xrightarrow{\varphi}_{\text{Borel}} \mathbb{R}$)
- $(\alpha_n \in M_X)_{n\in\mathbb{N}} \implies [r \in S_n.\alpha(r)] \in M_X$

s.t.:

$$(\alpha_n) \in M_X^{\omega} \implies \bigvee_n \alpha_n \in M_X$$

pointwise ω-chain: $(\alpha_n) \in M_X^{\omega}$

Borel measurable countable partition $\mathbb{R} = \biguplus_{n\in\mathbb{N}} S_n$

pointwise lub: $\bigvee_n \alpha_n$

Morphisms $f : X \to Y$: Scott continuous qbs maps

Scott continuous: monotone and $f \bigvee_n x_n = \bigvee_n f\, x_n$

qbs maps: $\forall \alpha \in M_X.\ f \circ \alpha \in M_Y$

Quasi-Borel pre-domains

Example

$S = (\underline{S}, \Sigma_S)$ measurable space

$$(\underline{S}, =, \{\alpha : \mathbb{R} \to \underline{S} | \alpha \text{ Borel measurable}\})$$

so $\mathbb{R} \in \omega\mathbf{Qbs}$

Reminder

wqbs: $X = (\underline{X}, \leq_X, M_X)$

- $\lambda\_.x \in M_X$
- $\alpha \in M_X \implies \alpha \circ \varphi \in M_X$
- $(\alpha_n \in M_X)_{n \in \mathbb{N}} \implies [r \in S_n.\alpha(r)] \in M_X$

s.t.:

$$(\alpha_n) \in M_X^{\omega} \implies \bigvee_n \alpha_n \in M_X$$

Quasi-Borel pre-domains

Example

$P = (\underline{P}, \leq_P)$ ω-cpo

lubs of
step functions

$$\left(\underline{P}, \leq_P, \left\{ \bigvee_k [\_ \in S_n^k . a_n^k] \middle| \forall k . \mathbb{R} = \biguplus_n S_n^k \right\}\right)$$

so $\mathbb{L} = ([0,\infty], \leq, \{\alpha : \mathbb{R} \to [0,\infty] | \alpha$ Borel measurable$\}) \in \omega\mathbf{Qbs}$

Reminder

wqbs: $X = (\underline{X}, \leq_X, M_X)$

- $\lambda \_ . x \in M_X$
- $\alpha \in M_X \implies \alpha \circ \varphi \in M_X$
- $(\alpha_n \in M_X)_{n \in \mathbb{N}} \implies [r \in S_n . \alpha(r)] \in M_X$

s.t.:

$$(\alpha_n) \in M_X^\omega \quad \implies \quad \bigvee_n \alpha_n \in M_X$$

Quasi-Borel pre-domains

Example

X ω-qbs

$$X_\perp := \left(\{\perp\} + \underline{X}, \perp \leq \underline{X}, \left\{[S.\perp, S^{\complement}.\alpha] \middle| \alpha \in M_X, S \text{ Borel}\right\}\right)$$

Reminder

wqbs: $X = (\underline{X}, \leq_X, M_X)$

- $\lambda\_.x \in M_X$
- $\alpha \in M_X \implies \alpha \circ \varphi \in M_X$
- $(\alpha_n \in M_X)_{n \in \mathbb{N}} \implies [r \in S_n.\alpha(r)] \in M_X$

s.t.:

$$(\alpha_n) \in M_X^\omega \qquad \implies \qquad \bigvee_n \alpha_n \in M_X$$

Quasi-Borel pre-domains

Products

$$\underline{X_1 \times X_2} = \underline{X}_1 \times \underline{X}_2 \qquad x \leq y \iff \forall i. x_i \leq y_i$$

$$M_{X_1 \times X_2} = \{(\alpha_1, \alpha_2) : \mathbb{R} \to \underline{X}_1 \times \underline{X}_2 | \forall i. \alpha_i \in M_{X_i}\}$$

correlated
random elements

Quasi-Borel pre-domains

Products

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correlated
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Theorem

$\omega\mathbf{Qbs} \to \omega\mathbf{Cpo} \times \mathbf{Qbs}$ *creates limits*

Quasi-Borel pre-domains

Products

$$\underline{X_1 \times X_2} = \underline{X}_1 \times \underline{X}_2 \qquad x \leq y \iff \forall i. x_i \leq y_i$$

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correlated random elements

Exponentials

- $\underline{Y^X} = \{f : \underline{X} \to \underline{Y} | f$ Scott continuous qbs morphism$\}$
  $= \mathbf{Qbs}(X, Y)$
- $f \leq g \iff \forall x \in \underline{X}. f(x) \leq g(x)$
- $M_{Y^X} = \left\{ \alpha : \mathbb{R} \to \underline{Y^X} \middle| \begin{array}{l} \mathrm{uncurry}\, \alpha : \mathbb{R} \times X \to Y \\ \text{Scott continuous qbs morphism} \end{array} \right\}$

so $\underline{Y}^{\mathbb{R}} = M_Y$

Fundamentals of measure theory

s-finite measures

- μ **bounded**: $\mu(\mathbb{R}) < \infty$
- μ **s-finite**: $\mu = \sum_n \mu_n$, μ_n bounded

Randomisation Theorem

Every s-finite measure is a push-forward of Lebesgue:

$$\mu \text{ s-finite} \implies \mu = f_* \lambda \text{ for some } f : \mathbb{R} \to \mathbb{R}_\perp$$

Transfer principle

$\tau_* \lambda = \lambda \otimes \lambda$ for some measurable $\tau : \mathbb{R} \xrightarrow{\cong} \mathbb{R} \times \mathbb{R}$

Randomisation monad structure

- $(X_\perp)^{\mathbb{R}}$
- $\mathrm{return}_X : r \in [0,1] \mapsto x$
- $(\alpha \gg= f) : \mathbb{R} \xrightarrow{\tau} \mathbb{R} \times \mathbb{R} \rightharpoonup \alpha \times \mathrm{id} \xrightarrow{\mathrm{eval} f \times \mathrm{id}} (Y_\perp)^{\mathbb{R}} \times \mathbb{R} \xrightarrow{\mathrm{eval}} Y$

$\mathbb{R} \to X_\perp$ $\qquad$ $X \to (X_\perp)^{\mathbb{R}}$

monad laws fail
(associativity)

Lebesgue integration

$$\alpha \begin{smallmatrix}\mathbb{R}\\ \downarrow \\ X_\perp\end{smallmatrix}, f \begin{smallmatrix}X\\ \downarrow \\ \mathbb{L}\end{smallmatrix} \mapsto \quad f \circ \alpha \begin{smallmatrix}\mathrm{Dom}(\alpha)\\ \downarrow_{\text{Borel}} \\ \mathbb{L}\end{smallmatrix} \quad \mapsto \int_{\mathrm{Dom}(\alpha)} f \circ \alpha(x) \lambda(\mathrm{d}x)$$

Lebesgue measure

$\mathrm{Dom}(\alpha)$ Borel

Borel maps and natural order

$$\int : (X_\perp)^{\mathbb{R}} \longrightarrow \mathbb{L}^{\mathbb{L}^X}$$

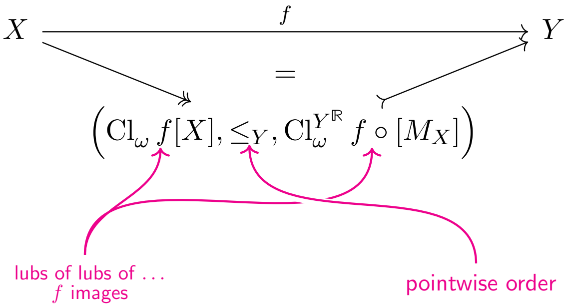
preserves monad structure

continuation monad

$$\begin{array}{ccc} (X_\perp)^{\mathbb{R}} & \xrightarrow{\quad\int\quad} & \mathbb{L}^{\mathbb{L}^X} \\ & \searrow\!\!\!\!\!\!\rightarrow \quad = \quad \nearrow & \\ & MX & \end{array}$$

MX: randomisable integration operators

A probabilistic powerdomain



$(\mathcal{E}, \mathcal{M}) :=$ (densely strong epi, full mono) factorisation system

A probabilistic powerdomain

$\mathcal{E} =$ densely strong epis closed under:

- products: $e_1, e_2 \in \mathcal{E} \implies e_1 \times e_2 \in \mathcal{E}$
- lifting: $e \in \mathcal{E} \implies e_\perp \in \mathcal{E}$
- random elements: $e \in \mathcal{E} \implies e^{\mathbb{R}} \in \mathcal{E}$

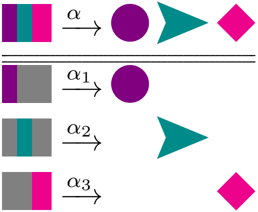
$\implies$ M strong monad for sampling + conditioning

[Kammar-McDermott'18]

A probabilistic powerdomain

$$\begin{array}{ccc} (X_\perp)^{\mathbb{R}} & \longrightarrow & \mathbb{L}^{\mathbb{L}^X} \\ & \searrow \;=\; \nearrow & \\ & MX & \end{array}$$

- M locally continuous $\implies$ may appear in domain equations
- M commutative $\implies$ satisfies Fubini
- M models synthetic measure theory
  $M \sum_{n \in \mathbb{N}} X_n \cong \prod_{n \in \mathbb{N}} MX_n$



[Kock'12, Ścibior et al.'18]

- $MX \cong \left\{ \mu|_{\text{Scott opens}} \,\middle|\, \mu \text{ is s-finite} \right\}$ generalises valuations

standard Borel space

Axiomatic domain theory

[Fiore-Plotkin'94, Fiore'96]

Structure

- Total map category: $\omega\mathbf{Qbs}$
- Admissible monos: **Borel-open** map $m : X \rightarrowtail Y$:

strong mono

qbses

$$\forall \beta \in M_Y. \qquad \beta^{-1}[m[X]] \in \mathcal{B}(\mathbb{R})$$

  take Borel-Scott open maps as admissible monos
- $\mathbf{Pos}$-enrichment: pointwise order
- Pointed monad on total maps: the powerdomain

$\Longrightarrow$ model axiomatic domain theory

$\Longrightarrow$ solve recursive domain equations

Axiomatic domain theory

Structure

$\mathfrak{D}$	total map category $\omega\mathbf{Qbs}$
$f \leq g$	$\mathbf{Pos}$-enrichment pointwise order
$\mathcal{M}_{\mathfrak{D}}$	admissible monos Borel-Scott opens
T	monad for effects power-domain
m	partiality encoding $m : -_{\perp} \to T,\ \perp \mapsto \underline{0}$

Axioms

$(\dashv)$	every object has a partial map classifier $\downarrow_X : X \to X_{\perp}$
(fup)	every admissible mono is full and upper-closed
$(\dashv_{\leq})$	$\lfloor - \rfloor$ is locally monotone
$(\bigvee)$	$\mathfrak{D}$ is $\omega\mathbf{Cpo}$-enriched
(U)	ω-colimits behave uniformly
$(\mathbb{1})$	$\mathfrak{D}$ has a terminal object
$(\to_{\leq})$	$\mathfrak{D}$ has locally monotone exponentials
$(+)$	locally continuous total coproducts
$(?!)$	$\mathbb{0} \to \mathbb{1}$ is admissible
$(\times_{\bigvee})$	$\mathfrak{D}$ has a locally continuous products
(CL)	$\mathfrak{D}$ is cocomplete
$(T_{\bigvee})$	T is locally continuous

Derived axioms/structure

$\mathbf{p}\mathfrak{D}$	partial map category
$-_{\perp}$	partiality monad
$(\dashv_{\bigvee})$	the adjunction $J \dashv L$ is locally continuous
$(\mathbf{p}_{\bigvee})$	$\mathbf{p}\mathfrak{D}$ is $\omega\mathbf{Cpo}$-enriched
$(\mathbb{1}_{\leq})$	$\mathbf{p}\mathfrak{D}$ has a partial terminal
$(\otimes)$	$\mathbf{p}\mathfrak{D}$ has partial products
$(\otimes_{\bigvee})$	$(\otimes)$ is locally continuous
$(\to_{\bigvee})$	$\mathfrak{D}$ has locally continuous exponentials
$(\rightharpoonup_{\bigvee})$	$\mathbf{p}\mathfrak{D}$ has locally continuous partial exponentials
$(\mathbf{p}CL)$	$\mathbf{p}\mathfrak{D}$ is cocomplete
$(\mathbf{p}+_{\bigvee})$	$\mathbf{p}\mathfrak{D}$ has locally continuous partial coproducts
(BC)	$J : \hookrightarrow \mathbf{p}\mathfrak{D}$ is a bilimit compact expansion

Summary

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- $\omega\mathbf{Qbs}$: a category of pre-domain quasi-Borel spaces
- M: commutative probabilistic powerdomain over $\omega\mathbf{Qbs}$

Theorem (adequacy)

M *adequately interprets:*

- *Statistical FPC*
- *Untyped Statistical λ-calculus*

This talk

- $\omega\mathbf{Qbs}$
- a powerdomain over $\omega\mathbf{Qbs}$
- a domain theory for $\omega\mathbf{Qbs}$

Not in this talk

- Operational semantics à la Borgström et al. ['16]
- Characterising $\omega\mathbf{Qbs}$