A syllabus for algebraic effects

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Learning journey

Basic definitions

Inaccessible literature: filtered colimits, monadicity, locally presentable categories, adjoint functor theorems, Lawvere theories, ...

> satisfaction in a Σ-algebra, obtaining the notion of a (Σ, E)-algebra in C. This, with the evident definition of homomorphism of algebras, generates a category (Σ, E)-Alg with a forgetful functor
>
> $$U : (\Sigma, E) - Alg \longrightarrow C$$
>
> which, if C is locally presentable, has a left adjoint F, inducing a monad $T = UF$ on C. The category (Σ, E)-Alg is isomorphic to the category T-Alg of algebras for the monad T.

– Plotkin and Power, "Notions of computation determine monads", 1^{st} paragraph after the introduction

and later: powers and copowers, enrichment, presheaf categories, sketches, Kan extensions, nerve and dense functors, ...

Goal

Accessible semantics of algebraic effects

Roadmap: syllabus for graduate students

(Cambridge Computer Science MPhils)

Setting: target audience

Course format

Lecture class (9 lectures $= 2$ per week $\times 4\frac{1}{2}$ weeks)
50 minute lectures
(7 more lectures with Marcelo Fiore on abstract syntax with binding)

Attendees

5 students taking the class
2 students sitting in
5 PhDs and Postdocs

Not in this talk:

Evaluation, course material, pedagogy
(course under development!)

Design decisions

Work within and around **Set**

Keep (categorical) concepts concrete.
Rich toolkit (e.g., equational logic).

Focus on *semantics*, not categories

Rich categorical picture.
Maintain a computer science connection.

Convey semantic intuition

Obscured by mathematical apparatuses in literature.
Offer vantage points.

Setting: background

Secret to success: prerequisites

Andy Pitts

'Category Theory and Logic' module:

- categories
- products and equational logic
- exponentials, typed λ-calculus and CCCs
- functors
- naturality
- presheaves
- Yoneda
- pullbacks
- adjunctions

No domain theory!

as not taught everywhere :(

Bird's eye

Syllabus

1. Pure λ-calculus
2. Moggi's λ_c
3. Equational logic, universal algebra, and monads
4. Model construction
5. Language design
6. Effect combination
7. Type-and-effect systems
8. Effect handlers
9. Programmable handlers

Starting point

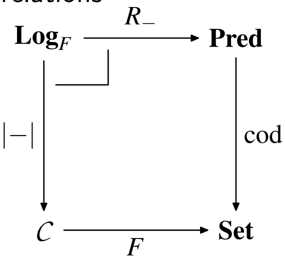
Simply-typed λ-calculus with sum types

Semantic concepts

- Equational theory
- CBV Felleisen SOS
- Denotational semantics
- Adequacy proof

Categorical concepts

- Distributive categories, bi-CCCs
- A category for logical relations



Rationale

- Mostly familiar
- Align baseline
- Modular logical relations

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Enter: effects

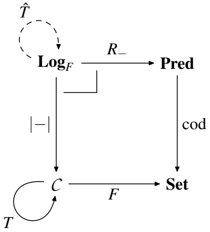
Moggi's λ_c

Semantic concepts

- Failure of equational theory
- Adequacy and the mono requirement
- Lack of general SOS

Categorical concepts

- Strong monads
- Lifting of a monad
- Hermida's lifting



Rationale

- Most have heard about Moggi/monads
- First brush against open problems

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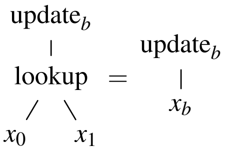
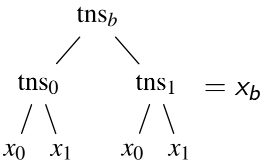
Algebraic interlude

Semantic concepts

- Computational models
  - exceptions
  - non-determinism
  - mnemoids



- Presentation sensitivity



test and set algebras

Mathematical concepts

- Review eq. logic
- Universal algebra
- Free model monad
- Unranked monads: Powerset, continuations

Rationale

- Effects as algebraic operations
- Algebraic manipulation of monads
- Limitations (rank)

Bird's eye

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Algebraic model design

Abstract device driver interaction

Semantic concepts

- Interface: $\mathit{lookup} : |\mathit{State}|$, $\mathit{act}_m : 1$
- Equations:

$$\begin{array}{c} act_{m_1} \\ | \\ act_{m_2} \\ | \\ x \end{array} = \begin{array}{c} act_{m_1 \cdot m_2} \\ | \\ x \end{array}$$

- How to choose the right monad?

Mathematical concepts

- Hilbert-Post completeness
- Monad calculation $\prod_{s \in \mathit{State}} sM \times -$
- Monoid actions + orbits as abstract automata

Rationale

- Non-obvious monad
- Open problem: model construction

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Algebraic language design

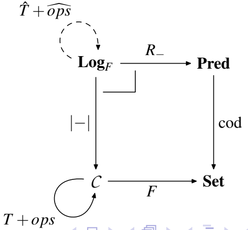
λ_{alg}: Algebraic lambda calculus

Semantics concepts

- λ_c + Kleisli arrows $a \to Tb$
- A closed language
- No SOS still

Categorical concepts

- Mention $\top\top$-lifting [Katsumata'05,'11]
- Algebraic lifting [Kammar'14]
- Generic effects and alg. operations $(TX)^b \to (TX)^a$



Rationale

- Semantically motivate (continuation-based) alg. operations
- General metalanguage for effects

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Sum and tensor

Semantic concepts

- Modular model/program construction
- Monad transformers composition order
- Graph tool

Rationale

- Still an open problem
- Haskell-relevant

Mathematical concepts

- Monads don't compose, e.g.:

$$\Big((1+1)\times(-)\Big)\circ(X\mapsto 1)$$

  is NaM (ta Conor)
- Monad transformers
- Sum and tensor
- Cographs

Bird's eye

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Model analysis

Type-and-effect systems

Semantic concepts

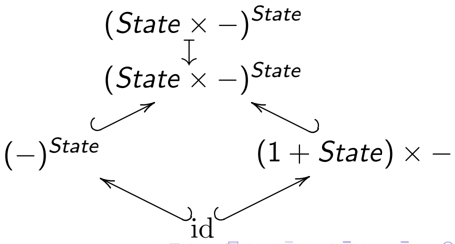
- Syntax and semantics
- Model generation
- Compiler transformation validation (soundness and completeness)

Mathematical concepts

- Monad morphisms
- Conservative extension/restriction
- Application to algebraic lifting

Rationale

- Solve an open problem
- Application area outside den. sem.
- For programmable handlers



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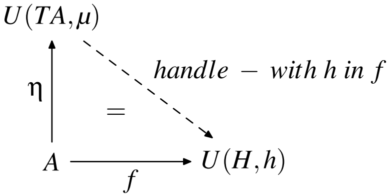
Effect deconstruction

Semantics for effect handlers

Semantic concepts

- 'handle' is not an alg. op.
- $\lambda_{alg}+$ fixed set of handlers
- equational laws for handlers [Plotkin & Pretnar'09]



Categorical concepts

- Algebras and homomorphisms for a monad

Rationale

- Incorporate exception handlers
- Handle non-free effects
- Possible for unranked monads

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Tying it all up

Programmable handlers

- λ_{eff}: user-defined alg. effects and handlers
- operational and denotational semantics
- programming examples

Rationale

- Synthesises:
  - (free) theories
  - effect systems
  - effect handlers,
  - algebraic lifting (for adequacy)
- "Hot" and active research topic

Conclusion

- A graduate-level syllabus
- Gateway to more advanced mathematical concepts
- Fits in half a lecture course
  (9 lectures), can co-exist with broader context
  (e.g., recursive domain equations).
- Inconclusive success
  (still under development)

Further work

- Course material, e.g.:
  lecture notes
  exercises
- Pedagogy

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Images

- http://cmseducation.org/syllabus/images/syllabus.gif
- http://www.mpi-sws.org/~dreyer/parametric/pitts.jpg